

AS Level Mathematics A

H230/01 Pure Mathematics and Statistics

Question Set 1

1 In this question you must show detailed reasoning.

- (i) Express $3^{\frac{7}{2}}$ in the form $a\sqrt{b}$, where a is an integer and b is a prime number. [2]

$$3^{\frac{7}{2}} = \sqrt{3^7} = \sqrt{2187} = 27\sqrt{3}$$

- (ii) Express $\frac{\sqrt{2}}{1-\sqrt{2}}$ in the form $c+d\sqrt{e}$, where c and d are integers and e is a prime number. [3]

$$\frac{\sqrt{2} \times \frac{1+\sqrt{2}}{1+\sqrt{2}}}{1-\sqrt{2}} = \frac{2+\sqrt{2}}{-1} = -2-\sqrt{2}$$

- 2 (i) The equation $x^2+3x+k=0$ has repeated roots. Find the value of the constant k . [2]

$$b^2-4ac=0 \quad 9-4k=0 \quad k=\frac{9}{4}$$

- (ii) Solve the inequality $6+x-x^2 > 0$. [2]

$$x^2-x-6 < 0 \quad -2 < x < 3$$

$$(x-3)(x+2)$$

- 3 (i) Solve the equation $\sin^2\theta = 0.25$ for $0^\circ \leq \theta < 360^\circ$. [3]

$$\sin\theta = \pm 0.5 \quad \theta = 30, 150, 210, 330$$

- (ii) In this question you must show detailed reasoning.

Solve the equation $\tan 3\phi = \sqrt{3}$ for $0^\circ \leq \phi < 90^\circ$. [3]

$$3\phi = 60 \quad \phi = 20 \quad 0 \leq \phi < 90$$

$$3\phi = 240 \quad \phi = 80 \quad 0 \leq 3\phi < 270$$

- 4 (i) It is given that $y = x^2 + 3x$.

- (a) Find $\frac{dy}{dx}$. [2]

$$= 2x+3$$

- (b) Find the values of x for which y is increasing. turning point at $x = -1.5$ is min. $\therefore x > -1.5$ [2]

- (ii) Find $\int (3-4\sqrt{x}) dx$. [5]

$$= 3x - \frac{8}{3} x^{\frac{3}{2}}$$

- 5 N is an integer that is not divisible by 3. Prove that N^2 is of the form $3p+1$, where p is an integer. [5]

$$N = 3p+1 \text{ or } 3p+2 \quad (3p+2)^2 = 9p^2+12p+4$$

$$(3p+1)^2 \therefore \text{in form } 3p+1 \quad = 9p^2+4(3p+1)$$

- 6 Sketch the following curves.

- (i) $y = \frac{2}{x}$ [2]

- (ii) $y = x^3 - 6x^2 + 9x$ [5]



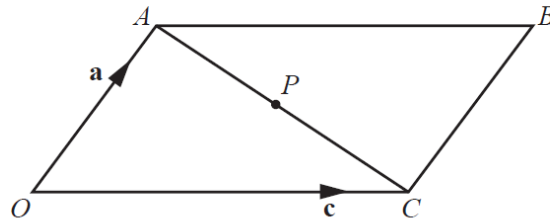
$$x(x-3)^2 \quad \frac{dy}{dx} = 3x^2 - 12x + 9$$

$$x^2 - 4x + 3 = 0$$

$$x = 3 \quad x = 1$$

turning point

7 $OABC$ is a parallelogram with $\vec{OA} = \mathbf{a}$ and $\vec{OC} = \mathbf{c}$. P is the midpoint of AC .



(i) Find the following in terms of \mathbf{a} and \mathbf{c} , simplifying your answers.

(a) $\vec{AC} = \mathbf{c} - \mathbf{a}$ [1]

(b) $\vec{OP} = \frac{1}{2}(\mathbf{a} + \mathbf{c})$ [2]

(ii) Hence prove that the diagonals of a parallelogram bisect one another. [4]

$\vec{OM} = \mu(\mathbf{a} + \mathbf{c})$ (where m is the midpoint of \vec{OC} and \vec{AB})
 $\vec{OA} + \vec{AM} = \mathbf{a} + \lambda(\mathbf{a} + \mathbf{c})$
 $\mu = 1 - \lambda \therefore \mu = \lambda = \frac{1}{2} \therefore$ bisect
 $\mu = \lambda$

8 In this question you must show detailed reasoning.

The lines $y = \frac{1}{2}x$ and $y = -\frac{1}{2}x$ are tangents to a circle at $(2, 1)$ and $(-2, 1)$ respectively. Find the equation of the circle in the form $x^2 + y^2 + ax + by + c = 0$, where a , b and c are constants. [6]

normals go through the circle centre

\therefore normals intersect at centre

$$-2x + 3 = 2x + 5$$

$$x = -0.5$$

$$y = 4$$

$$(x + 0.5)^2 + (y - 4)^2 = r^2$$

using $(2, 1)$

$$(2.5)^2 + 3^2 = r^2$$

$$r^2 = 15.25$$

$$x^2 + x + 0.25 + y^2 - 8y + 16 = 15.25$$

$$x^2 + x + y^2 - 8y + 1$$

Total Marks for Question Set 1: 49

equations of normals

$$y = -2x + c \rightarrow \text{goes through } (2, 1)$$

$$\therefore 1 = -2 + c \quad c = 3$$

$$y = -2x + 3$$

$$y = 2x + c \rightarrow \text{goes through } (-2, 1)$$

$$\therefore 1 = -4 + c \quad c = 5$$

$$y = 2x + 5$$

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