Oxford Cambridge and RSA

# AS Level Mathematics A

## **H230/01** Pure Mathematics and Statistics

# Question Set 1

**1** **In this question you must show detailed reasoning.**

**(i)** Express $3^{\frac{7}{2}}$ in the form $a\sqrt{b}$, where $a$ is an integer and $b$ is a prime number. **[2]**

$3^{\frac{7}{2}} = \sqrt{3^7} = \sqrt{2187} = 27\sqrt{3}$

**(ii)** Express $\dfrac{\sqrt{2}}{1-\sqrt{2}}$ in the form $c + d\sqrt{e}$, where $c$ and $d$ are integers and $e$ is a prime number. **[3]**

$\dfrac{\sqrt{2}}{1-\sqrt{2}} \times \dfrac{1+\sqrt{2}}{1+\sqrt{2}} = \dfrac{2+\sqrt{2}}{-1} = -2-\sqrt{2}$

**2** **(i)** The equation $x^2+3x+k=0$ has repeated roots. Find the value of the constant $k$. **[2]**

$b^2-4ac=0$ $\quad 9-4k=0$ $\quad k=\frac{9}{4}$

**(ii)** Solve the inequality $6+x-x^2>0$. **[2]**

$x^2-x-6<0$ $\quad -2<x<3$

$(x-3)(x+2)$

**3** **(i)** Solve the equation $\sin^2\theta = 0.25$ for $0° \leqslant \theta < 360°$. **[3]**

$\sin\theta = \pm 0.5$ $\quad \theta = 30, 150, 210, 330$

**(ii)** **In this question you must show detailed reasoning.**

Solve the equation $\tan 3\phi = \sqrt{3}$ for $0° \leqslant \phi < 90°$. **[3]**

$3\theta = 60$ $\quad \theta = 20$ $\qquad 0 \leqslant \phi < 90$

$3\theta = 240$ $\quad \theta = 80$ $\qquad 0 \leqslant 3\phi < 270$

**4** **(i)** It is given that $y = x^2+3x$.

**(a)** Find $\dfrac{dy}{dx}$. $= 2x+3$ **[2]**

**(b)** Find the values of $x$ for which $y$ is increasing. turning point at $x=-1.5$ is min $\therefore$ $x>-1.5$ **[2]**

**(ii)** Find $\int (3-4\sqrt{x})\,dx$. $= 3x - \frac{8}{3}x^{\frac{3}{2}}$ **[5]**

**5** $N$ is an integer that is not divisible by 3. Prove that $N^2$ is of the form $3p+1$, where $p$ is an integer. **[5]**

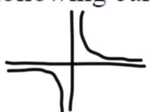
$N = 3p+1$ or $3p+2$ $\qquad (3p+2)^2 = 9p^2+12p+4$

$(3p+1)^2$ $\therefore$ in form $3p+1$ $\qquad = 9p^2+4(3p+1)$

$\therefore$ in form $3p+1$

**6** Sketch the following curves.

**(i)** $y = \dfrac{2}{x}$ **[2]**



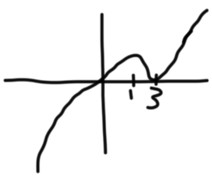
**(ii)** $y = x^3-6x^2+9x$ $\quad x(x-3)^2$ **[5]**

$\dfrac{dy}{dx} = 3x^2-12x+9$

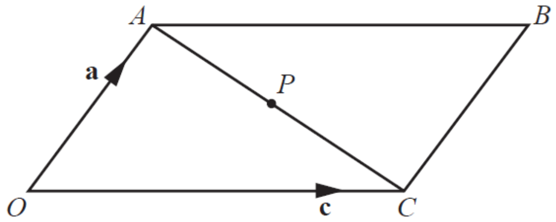
$x^2-4x+3=0$

$x=3$ $\quad x=1$

turning point



**7** $OABC$ is a parallelogram with $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$. $P$ is the midpoint of $AC$.




**(i)** Find the following in terms of **a** and **c**, simplifying your answers.

**(a)** $\overrightarrow{AC}$ $-\underline{a}+\underline{c}$ **[1]**

**(b)** $\overrightarrow{OP}$ $\frac{1}{2}(\underline{a}+\underline{c})$ **[2]**

**(ii)** Hence prove that the diagonals of a parallelogram bisect one another. **[4]**

$\overrightarrow{OM} = \mu(\underline{a}+\underline{c})$ (where m is the midpoint of $\overrightarrow{OC}$ and $\overrightarrow{AB}$)

$\overrightarrow{OA} + \overrightarrow{AM} = \underline{a} + \lambda(\underline{c}-\underline{a}+\underline{c})$

$\mu = 1-\lambda \therefore \mu = \lambda = \frac{1}{2} \therefore$ bisect

$\mu = \lambda$

**8** **In this question you must show detailed reasoning.**

The lines $y = \frac{1}{2}x$ and $y = -\frac{1}{2}x$ are tangents to a circle at (2, 1) and (−2, 1) respectively. Find the equation of the circle in the form $x^2 + y^2 + ax + by + c = 0$, where $a$, $b$ and $c$ are constants. **[6]**

normals go through the circle centre $\longrightarrow \therefore$ normals intersect at centre

**Total Marks for Question Set 1: 49**

equations of normals

$y = -2x + c \rightarrow$ goes through $(2,1)$

$\therefore 1 = -2 + c \quad c = 3$

$y = -2x + 3$

$y = 2x + c \rightarrow$ goes through $(-2,1)$

$\therefore 1 = -4 + c \quad c = 5$

$y = 2x + 5$

$-2x + 3 = 2x + 5$

$x = -0.5$

$y = 4$

$(x+0.5)^2 + (y-4)^2 = r^2$

using $(2,1)$

$(2.5)^2 + 3^2 = r^2$

$r^2 = 15.25$

$x^2 + x + 0.25 + y^2 - 8y + 16 = 15.25$

$x^2 + x + y^2 - 8y + 1$